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Laplace transforms method to solve the initial value problems for systems of linear differential-algebraic equations

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Abstract— The aim of work is to solve the initial value problems of systems of differential-algebraic equations either by transforming them to systems of differential equations or without. We use Laplace transform method to solve the initial value problems for systems of linear differential-algebraic equations with constant coefficients.

Keywords: differential-algebraic equations, initial value problems, Laplace transform method, constant coefficients.

1. INTRODUCTION:

THIS systems of differential-algebraic equations arise in many areas of science and engineering such as robotics, biomechanics, control theory, electrical engineering, and fluid dynamics, the desirability of working directly with systems of differential-algebraic equations has been recognized for over thirty years by scientists and engineers in several areas., [Colin B., 2001]. [Azizi S. and Ramachandran V., 2006] gave a new method for distributed simulation of differential-algebraic equation systems was developed based on purely decentralized sliding mode control. [Peter K. and Volker M.,2007] described characterization. of classes of singular linear differential-algebraic equation, Due to the large amount of computation and communication associated with large scale matrix inversion problems in the existing centralized approaches, [Johan S.2008] presented Optimal Control and Model Reduction of Nonlinear Differential-Algebraic Equation Models" Linköping studies in science and technology Dissertations, [Severiano G., 2008] described an interesting family of Runge-Kutta methods for stiff problems and differential-algebraic equations.[Hector V. 2014] presented LPSM method as a combination of the classic series method and a resummation method based on the Laplace and Pad´e transforms.[7,8,9] study on diverse application of Laplace Transformation in various field.

2. LAPLACE TRANSFORM METHOD:

Recall that the Laplace transform of a function f specified at t>0, denoted by L{f(t)} (or F(s)) is defined by:

$$L\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\,dt \tag{2.1}$$

where s is a complex number. Laplace transform of f exists if the above integral converges for some values of s, [Murry R., 1965] Moreover, it is easy to check that:

$$L\{f'(t)\} = sL\{f(t)\} - f(0).$$

In this section, we use Laplace transform method to solve the initial value problem that consists of systems of the first order linear ordinary differential equations with constant coefficients:

$$y'(t) = Ax(t) + By(t) + f(t), \quad t > 0 \tag{2.1.a}$$

together with the system of linear algebraic equations:

$$Cx(t)+Dy(t)+g(t)=0, \quad t\geq 0 \tag{2.1.b}$$

and the initial condition

$$y(0)=\alpha \tag{2.1.c}$$

where A, B, C, and D are $n\times n$ constant matrices, f and g are $n\times 1$ vector functions, α is $n\times 1$ known constant vector and x, y are the $n\times 1$ vector functions that must be determined.

To do this we consider the following two cases:

Case (1):

If x is a differentiable $n\times 1$ vector function then by differentiating eq.(2.1.b) with respect to t one can get:

$$Cx'(t)+Dy'(t)+g'(t)=0 \tag{2.2}$$

By substituting eq.(2.1.a) into eq.(2.2) one can have:

$$Cx'(t)+DAx(t)+DBy(t)+Df(t)+g'(t)=0.$$

Now, if C is a nonsingular matrix then the above equation can be rewritten as:

$$x'(t)+C^{-1}DAx(t)+C^{-1}DBy(t)+C^{-1}Df(t)+C^{-1}g'(t)=0 \tag{2.3.a}$$

But

$$y'(t)=Ax(t)+By(t)+f(t) \tag{2.3.b}$$

In this case the original system of differential-algebraic equations given by eq.'s(2.1.a)-(2.1.b) reduced to the system of the differential equations given by eq.'s(2.3.a)-(2.3.b). Moreover, by substituting t=0 in eq.(2.1.b) one can get:

Cx(0)+Dy(0)+g(0)=0

But, C^{-1} exists

and

$$y(0)=\alpha \tag{2.3.c}$$

thus

$$x(0)=C^{-1}[-g(0)-D\alpha] \tag{2.3.d}$$

Therefore the initial value problem given by eq.'s(2.1) reduces to the initial value problem given by eq.'s(2.3). This initial value problem can be solving by any suitable method, say Laplace transform method, and numerical method namely Euler method.

On the other hand, if C is a singular matrix, then by taking the Laplace transform of both sides of eq.(2.2) and eq.(2.1.a) one can get:

C[sL{x(t)}–x(0)]+D[sL{y(t)}–y(0)]+sL{g(t)}–g(0)=0

and

sL{y(t)}–y(0)=AL{x(t)}+BL{y(t)}+L{f(t)}.

respectively

But y(0)=α and Cx(0)=–g(0)–Dy(0)=–g(0)–Dα,

therefore the above equations becomes:

s CL{x(t)}+g(0)+Dα+ sDL{y(t)}–Dα +sL{g(t)}–g(0)=0

and

sL{y(t)} –α =AL{x(t)}+BL{y(t)}+L{f(t)}.

Hence, the above two equations can be rewritten in the matrix form:

$$\begin{bmatrix} sC & sD \\ A & B-sI \end{bmatrix}\begin{bmatrix} L\{x(t)\} \\ L\{y(t)\} \end{bmatrix}=\begin{bmatrix} -sL\{g(t)\} \\ -\alpha-L\{f(t)\} \end{bmatrix}.$$

where I is n×n identity matrix. If the above matrix is nonsingular for some values of s then

$$\begin{bmatrix} L\{x(t)\} \\ L\{y(t)\} \end{bmatrix}=\begin{bmatrix} sC & sD \\ A & B-sI \end{bmatrix}^{-1}\begin{bmatrix} -sL\{g(t)\} \\ -\alpha-L\{f(t)\} \end{bmatrix}$$

and by taking the inverse Laplace transform of L{x(t)} and L{y(t)} one can get x(t) and y(t) that satisfy eq.'s(2.1).

Case (2):

If x is a nondifferentiable n×1 vector function then the algebraic equation given by eq.(2.1.a) can not be transformed to an ordinary differential equations. In this case if C is a nonsingular matrix then:

$$x(t)=-C^{-1}Dy(t)-C^{-1}g(t) \tag{2.4}$$

By substituting the above equation in eq.(2.1.a) one can get

$$y'(t)=(-AC^{-1}D+B)y(t)+f(t)-AC^{-1}g(t)$$

this equation can be solved together with the initial condition given by eq.(2.1.c) by using Laplace transform method to get:

$$(sI+AC^{-1}D-B)L\{y(t)\}=L\{f(t)\}-AC^{-1}L\{g(t)\}+\alpha$$

Therefore

$$L\{y(t)\}=(sI+AC^{-1}D-B)^{-1}[L\{f(t)\}-AC^{-1}L\{g(t)\}+\alpha]$$

$$sI+AC^{-1}D-B$$

Provided that is a nonsingular matrix for some values of s. By taking the inverse Laplace transform of both sides of the above equation one can get y(t) which can be substituted in eq.(2.4) to get x(t).

On the other hand, if C is a singular matrix that by taking the Laplace transform of both sides of eq.(2.1.a) and eq.(2.1.b) one can get:

AL{x(t)}+(B–sI)L{y(t)}+L{f(t)}+α =0

and

CL{x(t)}+DL{y(t)}+L{g(t)}=0.

The above system of algebraic equations can be rewritten in the matrix form:

$$\begin{bmatrix} A & B-sI \\ C & D \end{bmatrix}\begin{bmatrix} L\{x(t)\} \\ L\{y(t)\} \end{bmatrix}=\begin{bmatrix} -\alpha-L\{f(t)\} \\ -L\{g(t)\} \end{bmatrix}$$

which has the solution

$$\begin{bmatrix} L\{x(t)\} \\ L\{y(t)\} \end{bmatrix}=\begin{bmatrix} A & B-sI \\ C & D \end{bmatrix}^{-1}\begin{bmatrix} -\alpha-L\{f(t)\} \\ -L\{g(t)\} \end{bmatrix}\begin{bmatrix} A & B-sI \\ C & D \end{bmatrix}$$

provided that is a nonsingular matrix for some values of s Therefore by taking the Laplace transform of both sides of the above equation one can obtain x(t) and y(t).

To illustrate this method we consider the following examples.

Example (2.1):

Consider the initial value problem that consists of system of the first order linear differential equations with constant coefficients:

$$\begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix}=\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}+\begin{bmatrix} 12 & 7 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}+\begin{bmatrix} -11+7t^3 \\ t-2t^2-t^3 \end{bmatrix} \quad (2.5.a)$$

together with the system of the algebraic equations:

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}+\begin{bmatrix} \frac{1}{6} & 4 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}+\begin{bmatrix} -1-3t^3+4t^2 \\ -t-t^3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2.5.b)$$

and the initial conditions:

$$\begin{bmatrix} y_1(0) \\ y_2(0) \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

We solve this example by using Laplace transform method. To do this let

$$C=\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \quad \text{then} \quad C^{-1}=\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}.$$

Therefore eq.(2.5.b) becomes:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}=\begin{bmatrix} \frac{-17}{6} & 4 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}+\begin{bmatrix} -1+4t^2+3t \\ -t-t^3 \end{bmatrix} \quad (2.6)$$

by substituting the above equations in eq (2.5.a) one can obtain:

$$\begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix}=\begin{bmatrix} 29 & -48 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}+\begin{bmatrix} 1+48t^2-29t \\ 2t-2t^2 \end{bmatrix}.$$

Then by taking the Laplace transform of both sides of the above system one can have:

$$\begin{bmatrix} s-29 & 48 \\ 0 & s-2 \end{bmatrix}\begin{bmatrix} L\{y_1(t)\} \\ L\{y_2(t)\} \end{bmatrix}=\begin{bmatrix} \frac{1}{s}+\frac{96}{s^3}-\frac{29}{s^2} \\ \frac{2}{s^2}-\frac{4}{s^3} \end{bmatrix}$$

Therefore

$$\begin{bmatrix} L\{y_1(t)\} \\ L\{y_2(t)\} \end{bmatrix} = \begin{bmatrix} s-29 & 48 \\ 0 & s-2 \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{s}+\frac{96}{s^3}-\frac{29}{s^2} \\ \frac{2}{s^2}-\frac{4}{s^3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{(s-29)}\left(\frac{1}{s}+\frac{96}{s^3}-\frac{29}{s^2}\right)-\frac{48}{(s-29)(s-2)}\left(\frac{2}{s^2}-\frac{4}{s^3}\right) \\ \frac{1}{(s-2)}\left(\frac{2}{s^2}-\frac{4}{s^3}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s^2} \\ \frac{2}{s^3} \end{bmatrix}.$$

Then by taking the Laplace inverse of both sides of the above system one can get:

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} t \\ t^2 \end{bmatrix}$$

By substituting the y1(t) and y2(t) in eq (2.6) one can have:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1-\frac{1}{6}t \\ t^3 \end{bmatrix}$$

Example (2.2):

Consider the initial value problem that consists of system of first order linear differential-algebraic equations with constant coefficients:

$$\begin{bmatrix} y_1'(t) \\ y_2'(t) \\ y_3'(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 4 & 0 & 6 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 2 & 0 & 4 \\ 7 & 1 & 2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} +$$

$$\begin{bmatrix} -\sin t + 2e^t - 4\cos t - 4 \\ -4\sin t - 6t^2 - t - 2\cos t - 7 \\ -2\sin t + e^t - 2t^2 + t \end{bmatrix} \qquad (2.7.a)$$

together with the system of the linear algebraic equations:

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 6 & -1 & 2 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} +$$

$$\begin{bmatrix} -2\sin t + e^t - 2t^2 - 2\cos t - 2 \\ -\sin t - 2t^2 - t - 4\cos t - 4 \\ -3\sin t + e^t - 4t^2 + t - 2\cos t - 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (2.7.b)$$

and the initial conditions:

$$\begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad (2.7.c)$$

We solve this system by using Laplace transform method. To do this Let:

$$C = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix},$$

then $|C|=0$ and hence C^{-1} does not exists.

By taking Laplace transform of both sides of eq.'s(2.7.a)-(2.7.b) and by using the initial conditions given by eq.(2.7.c) one can get:

$$s\begin{bmatrix} L\{y_1(t)\} \\ L\{y_2(t)\} \\ L\{y_3(t)\} \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 4 & 0 & 6 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} L\{x_1(t)\} \\ L\{x_2(t)\} \\ L\{x_3(t)\} \end{bmatrix} + \begin{bmatrix} 2 & 0 & 4 \\ 7 & 1 & 2 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} L\{y_1(t)\} \\ L\{y_2(t)\} \\ L\{y_3(t)\} \end{bmatrix} +$$

$$\begin{bmatrix} \frac{-1}{s^2+1} - \frac{4}{s} + \frac{2}{s-1} - \frac{4s}{s^2+1} \\ \frac{-7}{s} - \frac{4}{s^2+1} - \frac{12}{s^3} - \frac{1}{s^2} - \frac{2s}{s^2+1} \\ \frac{-2}{s^2+1} + \frac{1}{s-1} - \frac{4}{s^3} + \frac{1}{s^2} \end{bmatrix}$$, and

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}\begin{bmatrix} L\{x_1(t)\} \\ L\{x_2(t)\} \\ L\{x_3(t)\} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 6 & -1 & 2 \end{bmatrix}\begin{bmatrix} L\{y_1(t)\} \\ L\{y_2(t)\} \\ L\{y_3(t)\} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{-2}{s^2+1} + \frac{2}{s} - \frac{1}{s-1} + \frac{4}{s^3} + \frac{2s}{s^2+1} \\ \frac{-1}{s^2+1} + \frac{4}{s^3} + \frac{4}{s} + \frac{1}{s^2} + \frac{4s}{s^2+1} \\ \frac{3}{s^2+1} + \frac{6}{s} - \frac{1}{s-1} + \frac{8}{s^3} - \frac{1}{s^2} + \frac{2s}{s^2+1} \end{bmatrix}$$

Respectively

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 0 & 6 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} L\{x_1(t)\} \\ L\{x_2(t)\} \\ L\{x_3(t)\} \end{bmatrix} + \begin{bmatrix} 2-s & 0 & 4 \\ 7 & 1-s & 2 \\ 0 & -1 & -s \end{bmatrix}\begin{bmatrix} L\{y_1(t)\} \\ L\{y_2(t)\} \\ L\{y_3(t)\} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{s^2+1} + \frac{4}{s} - \frac{2}{s-1} + \frac{4s}{s^2+1} - 1 \\ \frac{4}{s^2+1} + \frac{7}{s} + \frac{1}{s^2} + \frac{12}{s^3} + \frac{2s}{s^2+1} - 1 \\ \frac{2}{s^2+1} - \frac{1}{s^2} + \frac{4}{s^3} - \frac{1}{s-1} - 1 \end{bmatrix}.$$

Hence, the above two equations can be rewritten in the matrix form:

$$\begin{bmatrix} 1 & 2 & 0 & 2-s & 0 & 4 \\ 4 & 0 & 6 & 7 & 1-s & 2 \\ 1 & 1 & 2 & 0 & -1 & -s \\ 2 & 1 & 2 & 1 & 0 & 2 \\ 1 & 0 & 2 & 3 & 1 & 4 \\ 3 & 1 & 4 & 6 & -1 & 2 \end{bmatrix}\begin{bmatrix} L\{x_1(t)\} \\ L\{x_2(t)\} \\ L\{x_3(t)\} \\ L\{y_1(t)\} \\ L\{y_2(t)\} \\ L\{y_3(t)\} \end{bmatrix} = \begin{bmatrix} \frac{1}{s^2+1} + \frac{4}{s} - \frac{2}{s-1} - \frac{4s}{s^2+1} - 1 \\ \frac{7}{s} + \frac{4}{s^2+1} + \frac{12}{s^3} + \frac{1}{s^2} + \frac{2s}{s^2+1} - 1 \\ \frac{2}{s^2+1} - \frac{1}{s-1} + \frac{4}{s^3} - \frac{1}{s^2} - 1 \\ \frac{2}{s^2+1} + \frac{2}{s} - \frac{1}{s-1} + \frac{4}{s^3} + \frac{2s}{s^2+1} \\ \frac{1}{s^2+1} + \frac{4}{s^3} + \frac{4}{s} + \frac{1}{s^2} + \frac{4s}{s^2+1} \\ \frac{3}{s^2+1} + \frac{6}{s} - \frac{1}{s-1} + \frac{8}{s^3} - \frac{1}{s^2} + \frac{2s}{s^2+1} \end{bmatrix}$$

Let

$$M = \begin{bmatrix} 1 & 2 & 0 & 2-s & 0 & 4 \\ 4 & 0 & 6 & 7 & 1-s & 2 \\ 1 & 1 & 2 & 0 & -1 & -s \\ 2 & 1 & 2 & 1 & 0 & 2 \\ 1 & 0 & 2 & 3 & 1 & 4 \\ 3 & 1 & 4 & 6 & -1 & 2 \end{bmatrix}$$

then it is easy to check that |M|=144–104s and hence $|M| \neq 0$ for $s \neq \frac{144}{104}$. In this case:

$$M^{-1} =$$

$$\begin{bmatrix} \frac{-(s^2+8s-12)}{2(13s-18)} & \frac{s^2-8s+8}{2(13s-18)} & \frac{s^2-13s+16}{13s-18} & \frac{s^3-2s^2+66s-88}{4(13s-18)} & \frac{s^3-12s^2-2s+24}{4(13s-18)} & \frac{-(s^3-2s^2-18s+24}{4(13s-18)} \\ \frac{s^2+11s-30}{2(13s-18)} & \frac{-(s^2-5s+20)}{2(13s-18)} & \frac{-(s^2-10s+13)}{13s-18} & \frac{-(s^3+s^2+18s-58)}{4(13s-18)} & \frac{-(s^3-9s^2+24s-30)}{4(13s-18)} & \frac{s^3+s^2-14s+42}{4(13s-18)} \\ \frac{s^2+2s}{4(13s-18)} & \frac{-(s^2-14s+12)}{4(13s-18)} & \frac{-(s^2-19s+24)}{2(13s-18)} & \frac{-(s^3-8s^2+60s-60)}{8(13s-18)} & \frac{-(s^3-18s^2+36)}{8(13s-18)} & \frac{s^3-8s^2-24s+36}{8(13s-18)} \\ \frac{s-6}{2(13s-18)} & \frac{-(s+4)}{2(13s-18)} & \frac{-(s-1)}{13s-18} & \frac{-(s^2+10s-26)}{4(13s-18)} & \frac{-(s^2-6)}{4(13s-18)} & \frac{s^2+10s-6}{4(13s-18)} \\ \frac{-(s+18)}{2(13s-18)} & \frac{s-24}{2(13s-18)} & \frac{s-3}{13s-18} & \frac{s^2+8s+30}{4(13s-18)} & \frac{s^2-2s+18}{4(13s-18)} & \frac{-(s^2+8s-54)}{4(13s-18)} \\ \frac{s+6}{2(13s-18)} & \frac{-(s-10)}{2(13s-18)} & \frac{-(s-2)}{13s-18} & \frac{-(s^2-4s+20)}{4(13s-18)} & \frac{-(s^2-14s+24)}{4(13s-18)} & \frac{s^2-4s-12}{4(13s-18)} \end{bmatrix}$$

Therefore M^{-1} exists for $s \neq \frac{18}{13}$ and $s \neq \frac{144}{140}$.

Hence

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$$\begin{bmatrix} L\{x_1(t)\} \\ L\{x_2(t)\} \\ L\{x_3(t)\} \\ L\{y_1(t)\} \\ L\{y_2(t)\} \\ L\{y_3(t)\} \end{bmatrix} = \begin{bmatrix} \frac{1}{s^2+1} \\ \frac{1}{s}-\frac{1}{s-1} \\ \frac{2}{s^3} \\ \frac{1}{s} \\ \frac{1}{s}-\frac{1}{s^2} \\ \frac{s}{s^2+1} \end{bmatrix}$$

and therefore

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} L^{-1}\left\{\frac{1}{s^2+1}\right\} \\ L^{-1}\left\{\frac{1}{s}-\frac{1}{s-1}\right\} \\ L^{-1}\left\{\frac{2}{s^3}\right\} \\ L^{-1}\left\{\frac{1}{s}\right\} \\ L^{-1}\left\{\frac{1}{s}-\frac{1}{s^2}\right\} \\ L^{-1}\left\{\frac{s}{s^2+1}\right\} \end{bmatrix} = \begin{bmatrix} \sin t \\ 1-e^t \\ t^2 \\ 1 \\ 1+t \\ \cos t \end{bmatrix}$$

is the solution of the initial value problem given by eq.'s(2.7).

Conclusions and Recommendations:

From our work, it is convenient to mention that:

The Laplace transform are very active methods when it is applied to solve the eigenvalue problems associated with the linear differential-algebraic equations these methods can be also applied to solve the eigenvalue problems related to the nonlinear differential-algebraic equations.

Also, for future work, one can introduce the following problems:

Find another method for solving the generalized linear eigenvalue problems in partial differential-algebraic equations and make a comparison between these methods and the other methods which appeared in this work.

Use another type of functions instead of the polynomial to approximate the solution of the linear and nonlinear differential-algebraic equations of the first, second and third kinds.Use the same methods to solve the eigenvalue problems of the linear and nonlinear partial differential-algebraic equations in which the unknown function depend on more than two variables.

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